TWO MEANINGS OF THE 'EQUAL' SIGN AND SENSES OF COMPARISON AND SUBSTITUTION METHODS

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In this paper we analyze the meanings of the 'equal' sign as generated by sense production of the methods of substitution and comparison for solving problems and systems of equations of two unknowns. These methods are usually introduced through an extension process of the syntax and meanings recently learned by students in order to solve problems using linear equations with one unknown. Through this process some users were able to confer sense to the methods and thus generate the new meanings required.

In Filloy (1991) we introduced the notions of ***meaning*** and ***sense*** for analyzing the learning processes and the creation of rules which allow to coordinate the actions performed for solving one-unknown problems through "concrete models" (see Filloy and Rojano, 2001; and Filloy, Rojano and Solares, 2002). In this paper we use these notions for studying the transition from one-unknown representation and manipulation to the representation and manipulation of one unknown given in terms of another unknown—In fact, this transition corresponds to a ***didactic cut*** (see Filloy and Rojano, 1989; Solares, 2002). The new representation of the unknown is used in the ***comparison and substitution methods*** in such a way which allows the **reduction** of a two-unknown problem to a one-unknown problem and making possible to apply the previously learned syntax in order to solve one-unknown linear equations. In the particular case of the system $(S_1)\begin{cases} y = 12 - x \\ 5x - 6 = y \end{cases}$ the performance of the ***comparison method*** entails the equalization of two operation chains for one of the unknowns and for the data which allow to calculate the other unknown's value. That is, two ways for calculating the value of one of the unknowns are equalized. And in the case of the system $(S_2)\begin{cases} x + y = 12 \\ 5x - 6 = y \end{cases}$ to perform the ***substitution*** entails replacing the '***y***' in the second equation in the first and through this operation chain find the '***x***' value. Thus, a chain of operations is substituted into another.

We will analyze the meanings of the 'equal' sign generated by children between 13 and 14 years old when using the comparison and substitution methods in two-unknown equations' solving process. For the students interviewed the ***sense*** of this methods is given by the linking of all actions performed. At the beginning of the learning process these action chains are not yet provided of sense. The increasing syntactical complexity of the relations between the data and the unknowns, the changes in the data's or solutions' numerical domains, for example, obstruct both the use of the methods and the spontaneous solution strategies. At that stage, readings from more concrete strata of the new Mathematical Sign System do not allow to identify the changes in the problematic situation as members of the same kind of problems. Only when the sense conferred through the sequence of mathematical texts in the Teaching Model is acquired these strata will be identified as members of the same kind of problems –susceptible of being

solved through the same process, or chain of actions. That is the moment when the new notions –such as the new notion of equality– will become stable (see Matz, 1980; Kieran, 1981; Kieran and Sfard, 1999; Drouhard, 1992).

Here, it is useful to see once again the way in which Mt. (Filloy and Rojano, 1989, pp. 21-22), one of the subjects interviewed, generated the meaning assigned to the 'equal' sign when learning the syntax for solving one-unknown linear equations: **IMt26. 10x – 18 = 4x + 6...**

> **Mt:**...if I obtain the value of ***x*** and I perform that operation (points towards "***10x – 18***") I obtain one result. That result has to be equal to this (points towards "***4x + 6***")

THEORETICAL AND METHODOLOGICAL FRAMEWORK

For the experimental design a **Local Theoretical Model** (Filloy, 1990) was built up in order to explain, upon the semiotic notion of **Mathematical Sign Systems** (MSS) the empirical observations obtained through videotaped clinical interviews.

From the theoretical perspective of the **Local Theoretical Models** each specific object of study is analyzed through four interrelated components: (1) the **Formal Competence Model**; (2) the **Teaching Model**; and (3) the **Cognitive Process** and (4) **Communication Models**. Below, the specific characteristics of these components in our study will be described.

FORMAL COMPETENCE MODEL

In order to construct **the Formal Model** component, we used the syntax model for simple algebraic expressions and equations developed by Kirshner (1987) and completed by Drohuard (1992). Besides, we incorporated the semantic elements proposed by Drohuard (1992) in order to study the meanings of the algebraic writing. These studies on algebraic syntax and semantics render important results for teaching –such as Drouhard's definition of *automathe formel* for defining subjects who center their attention on the rules that have to be applied (*sens*) and not in the results obtained (*dénotation*)–, these studies do not incorporate to the analysis the spontaneous usage that learners give to already learned elements of the algebraic language in order to solve new problems.

The **Formal Competence Model** that we designed allows us to study the syntactical complexity of the algebraic substitution and comparison methods used for solving equation systems. Substitution method results more complex.

TEACHING MODEL

Upon the base of the analysis performed at the formal level we adopted the following **didactical route** for introducing these methods –coming from the previously acquired competencies for solving one-unknown linear equations: (1) reduction of the two-unknown and two-equation system to a one-unknown equation through the application of comparison or substitution; (2) solution of the one-unknown equation applying the previously learned syntax; (3) substitution of the numerical value found in one of the two equations; and (4) solution of the equation through the application of the previously learned syntax.