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the same we have to take it into consideration. For example, suppose the daily consumption of bakery A is 10 tons, and the daily consumption of bakery B is 5 tons. The relation between them is 10:5 that means 2:1. Therefore, the storeroom should be built $\frac{2}{3}$ of the distance between the bakeries, closer to the one that uses the most flour." The pupil based it on an argument that seems correct: the relation between the distances from the bakeries to the storeroom has to be opposite to the relation between the consumptions. But this argument is unsuitable for the given situation.

This phenomenon originated in the fact that there was no transfer to a completely mathematical model.

In conclusion, we would like to point out that the construction of a mathematical model that suits the problem phrased as an everyday situation was a necessary condition for solving the problem. Not one of the students succeeded in reaching a correct solution without first rephrasing the problem in mathematical terms, (partly or wholly). Therefore it is necessary to work on this stage with the pupils and to make them conscious of it.

The most common mistake in solving problems of the above kind was made by students basing themselves on mathematical arguments that did not suit the situation described. This happened because there was no transfer to a complete mathematical model. Another common mistake was the inclination to a practical solution with, or even without, a connection to what the problem demanded. Such a mistake was typical of pupils who made no attempt to rephrase the problem in mathematical terms.

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OPERATING THE UNKNOWN AND MODELS OF TEACHING (A Clinical Study Among Children 12-13 with High Proficiency in Pre-Algebra)

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ABSTRACT. Operating the unknown is one of the first properly algebraic actions. As reported in a former paper [2] operating arithmetical operations to new objects (e.g., the unknown) is not a spontaneous process in children, and requires special attention in the field of teaching. In this paper, we are reporting the most relevant results from the research work "Operating the Unknown" which refers to the constant, as well as to the variable aspects that were observed when using various models for operating the unknown in the resolution of linear equations having the form: $Ax + B = Cx + D$, where, A, B, C and D are given positive integers and $D \geq 0$. Observations were performed through clinical interviews to children 12-13 who had had no previous instruction in algebra, but had showed a high proficiency in pre-algebra. By way of conclusion, our study shows that all teaching strategies should contemplate a dialectic process between the most concrete meanings and the operational syntax, because both aspects are necessary to the students' anticipatory mechanisms, even if they have not been used at the time of devising the didactic situations (it occurs in the use of teaching strategies).

INTRODUCTION. It has always been thought that more concrete models possess the virtue of providing more stable meanings to new concepts, whereas more syntactic models have a tendency to introduce a certain senselessness to everything new that is taught. It is also commonly believed that the latter aspect is the one that contributes most strongly to the lack of interest on the part of children towards the study of mathematics (at the affective level), and to the tendency to commit the typical and spontaneous errors (cognoscitive level) when operative abilities required by algebra come into play. The results of this paper, however, show that even models where meanings are taken from a more concrete language, will pose, when used as teaching strategies in the resolution of linear equations, the same problems; naturally, these problems will have particular manifestations depending on the model under consideration.

GENERAL FRAMEWORK AND SPECIFIC OBJECTIVES OF THIS WORK. "Operating the Unknown" is a part of the research program "The Acquisition of Algebraic Language", which has been developed since 1980 at the Sección Matemática Educativa and the Centro Escolar Hermanos Revueltas, in Mexico City. Probing into the difficulties involved both in the syntactic handling of algebra, and the utilization of algebra to solve problems, in relation with didactic phenomena that appear during the child's transit from arithmetical to algebraic thought, are among objectives of the wider research. This study, "Operating the Unknown", has the following specific aims: 1) to observe the spontaneous responses of children that are faced, for the first time with "non-arithmetical" equations (i.e., those where resolution demands operating the unknown, as, for instance, in some linear equation with more than one occurrence of the un-

