

Psychology of Mathematics Education

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OBSTRUCTIONS TO THE ACQUISITION OF ELEMENTAL ALGEBRAIC CONCEPTS AND TEACHING STRATEGIES

*E. Filloy / T. Rojano **

In this communication we give the results of the analysis of a series of clinical video taped interviews, undertaken with children of 12 and 13 years of age, with a high proficiency in pre-algebra and who have not received previous algebra teaching. During the interview they receive instruction in the solution of the so-called "first non-arithmetical equations" (linear equations with occurrences of the unknown on both sides of the equality), through the use of a "concrete" model to operate the unknown. Through the development of the interviews, processes of abstractions become clear. In these processes we detect phenomena such as: momentary loss of previous skills, the modification of the arithmetical idea of equation, the use of personal graphs (codes) to indicate the actions on the elements of the equation, the 'rooting' to the model and the detachment from the model, obstructions due to the model itself, and general behaviour patterns common to all models, and the recognition through the models of the diversity of first degree equations types.

INTRODUCTION.- As mentioned in a previous study [4], operating the unknown is one of the first incursions into the territory of symbolic algebra. The stage of transition from arithmetic to algebra is crucial in the construction of algebraic language, in particular as regards the presence at this stage of obstacles closely related to the 'rooting' behaviours to ideas and arithmetical habits. (See - Filloy / Rojano [3]).

Recent studies have been directed to the detection and description of psychological and didactic phenomena occurring during this transition. (See Kieran [6], Bernard / Bright [1], Booth [2], Harper [5], Wagner [7] and Filloy / Rojano [4] for some examples).

In this paper, we are reporting relevant results from the research work "Operating the unknown", which refer to the constant, as well as to the variable aspects that were observed when using a geometrical model for operating the unknown in the resolution of linear equations having the form $Ax \pm B = Cx \pm D$, where A, B, C and D are given positive integers, and $D \geq 0$.

This research is encompassed in a study of wider scope, "The Acquisition of Algebraic Language" and both have been developed within the framework of the "Evolution of Symbolization Among a Middle-Basic Level School Population" Project, at the Sección de Matemática Educativa and the Centro Escolar Hermanos Revueltas, in Mexico City. Other research stages which have been described in the paper "From an Arithmetical to an Algebraic Thought" [4], provide a background to this clinical observation.

The clinical study covered two generations of High School students of one secondary school throughout the years 82-83 and 83-84, respectively. Both interviewing periods were preceded by written diagnoses for measuring pre-algebraic proficiency. On the basis of the results

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