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FROM AN ARITHMETICAL TO AN ALGEBRAIC THOUGHT
(A clinical study with 12-13 year olds)

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Abstract.- Theoretical and historical considerations seem to indicate that there is a didactical cut in the evolution line that goes from an arithmetical to an algebraic thought [2]. There are several aspects of the individual way of thinking which suffer essential changes when passing through this cut. Some of these aspects have been pointed out by C. Kieran [4]. We have carried out a clinical study with a stratified sample of 12-13 year olds that reveals some other aspects and previous difficulties to the acquisition of the algebraic language, for instance, that the children's tendency to stay at an arithmetical level when solving the first "non-arithmetical equations" (linear equations with two occurrences of the unkown) makes manifest certain behaviors related with their reading and interpretation of literal notation, algebraic symbols and algebraic expressions and with their solving strategies choice which would have necessarily to be taken into consideration in any teaching approach of elementary algebra.

INTRODUCTION.- The elementary algebraic language has to be constructed in child on arithmetical basis; this construction will be possible from extending the elementary arithmetical operations to new other objects (for example, to the unkown and whole algebraic expressions) but, at the same time, breaking with previous arithmetical notions (for example, with the notion of equality). Both, the generalized arithmetics and the new algebraic conceptions begins to be developed in child, by means of teaching, in a moment that we define as a didactical cut, [see Filloy/Rojano, 1983] , it is, the moment when child faces for the first time linear equations with occurrences of the unknown in both sides of the sign " = ". The purpose of the present study is to make clinical observations in children, in the moment of the cut, analyzing their spontaneous responses to the new kind of equations and correlated problems, and trying to detect those permanence behaviors in arithmetics which could be real obstacles for the algebraic language construction. The clinical study and its results are described below.

PREPARATION OF THE STUDY

The clinical study covered two generations of High School students of one secondary school throughout the years 82-83 and 83-84, respectively. Both interviewing periods were preceded by written diagnoses for measuring pre-algebraic efficiency. On the basis of the results therefrom obtained, the student population was stratified and a stratified sample was chosen for the clinical interview.

The theoretical considerations exposed in [2] led us to use the diagnostic test to determine the population's background on the three pre-algebraic subtopics, namely: i) literal notation equations, that is linear equations of the form X ± A = B, A x (X ± B) = C, AX = B (A, B and C being natural numbers); and ii) non literal notation equations, for example, □ ± A = B, A x □ = B (A and B, natural numbers); and iii) Semantics, that is, translation into equations, interpretation and/ or solution of statements containing problems corresponding to equations i) and ii) and to "first non-arithmetical equations", that is those in which the unknown (Ax + B = Cx; A, B and C being natural numbers) already occurs twice. A specific stratification of the 82-83 generation was carried out for each subtopic on the basis of quantitative and qualitative criteria (according to type and number of items solved), which resulted in five strata for each subtopic. This time, the sample was constituted by children who had been consistently placed in one of the strata according to the three subtopics (see figure 1); only the extreme strata (1 and 5) and the intermediate stratum (3) were considered (figure 1, shaded areas).

	stratum	AXIS 1	AXIS 2	AXIS 3
high	5			6
	4			
middle	3			4
	2			
low	1			4

Fig 1

Thus, 6 children for the upper stratum, 4 from the intermediate one and 4 from the lower one were interviewed.

In the 83-84 test, the same subtopics were considered: i), ii) and iii). This time, a linear stratification for each subtopic or axis was also carried out —four strata fitting into each axis – and a spatial stratification was subsequently carried out in which each child was assigned an ordered triplet the coordinates of which correspond to the efficiency level in each axis, and is shown in a cube as the one in figure 2.

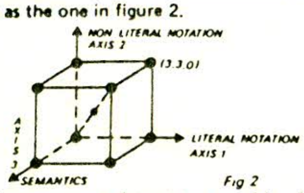


Fig 2

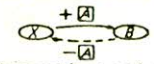
The sample was made up considering mainly the strata or classes along the principal diagonal and the vertices of the cube.

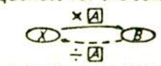
The two year samples are comparable, since the classes distributed along the principal diagonal correspond (given the fact that analogous criteria were applied) to strata shown in sample 82-83. The main difference roots off from the fact that, in 83-84, cases countermining the order of one or more axis were taken into consideration, for example, triplets (0, 0, 3), (3, 3, 0), (0, 3, 3), etc. Thus, both diagnoses cast a strata based view of the situation of the population surveyed, just at the moment which we have called didactical cut in the transition process from arithmetics to algebra, during which the interviews were carried out.

SCHOOL TREATMENT PRECEDING THE STUDY

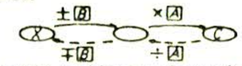
As we mentioned before, we have worked with the population of a secondary school in Mexico City; here, the material used in teaching mathematics has been made up by a group of teachers from the Sección Matemática Educativa [1]. As regards second grade Arithmetics and

Algebra, this material intends to teach the student, first of all, solutions to the simplest equatic (X ± A = B y Ax = B), as well as to problems arising therefrom, through the use of a sche representing direct operation and inverse operation sequences for the solution.





From there we go on to equations and problems requiring two or more operations: Ax ± B = A x (X±B) = C, using the same device.



Once this has been solved, we go on to problems and equations of the type X ± A = BX, that c only be solved by operating the unknown. We consider this the beginning of Algebra prope speaking, since it differs from Arithmetics in at least two aspects: on the one hand, the solvi method requiring only arithmetical operativity with numbers —mainly concerning the relatic ship between elementary operations and their inverses— now transfers such operativity to the known (which up to now is only the representation of an unknown number); on the other har the equal sign relating arithmetical operations (left side) to their result (right side) acquires nov "more algebraic" connotation, since the unknown occurs on both sides and therefore the expr sions which the equal sign links are more general and "equivalent" [4] . Traditionally, t transition is not taken into consideration in teaching and, thus, the arithmetical method is exter ed to more general cases disregarding the difficulties that the students may face in making su transference. The present study is to put forward such considerations.

The material for a clinical interview was prepared in order to observe the children's spontaneo attitudes when facing for the first time this new type of equations where the unknown must operated, or which require a more evolved notion of equality than the arithmetical one, example to procede to cancellation strategies.

CLINICAL INTERVIEW

It includes five different series of items: 1) E equation series, for diagnosis verification; this ser includes equations of the form X ± A = B, A = B, A x (X ±B) = C, and (X ± A) x B = C. 2 cancellation series, to determine the level of "conceptualization" of the equation as an equiv ence. It contains items such as: X ± A = B ± A, AX ± B = X ± B, $X \pm \frac{X}{A} = B \pm \frac{X}{A}$ and X+X= A +X . 3) Operating the unknown I series, in which we find items such as AX ± B = CX and AX ± B = CX ± D, to sound into students' difficulties when facing this new ty of equations. This series largely includes an instructional section given by the interviewer, l referring to a concrete context. 4) Number finding A series, to analyze the solving procedu in the different strata. This series includes two kinds of problems: those which may be solv through equations of the form X ±A=B, AX=B ó A x (X ±B) = C and those which can be solv with equations of the form AX ± B = CX. At this point, it is interesting to observe if the stud