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MATHEMATICAL MODELLING: THE INTERACTION OF CULTURE AND PRACTICE

ABSTRACT. Using a sociocultural approach we analyse the results of a Mexican/British project which investigated the ways in which mathematics is used in the practice of school science and the role of spreadsheets as a mathematical modelling tool. After discussing the different school cultures experienced by two groups of pre-university 16–18 year old students we analyse how these different cultures influenced their practice of mathematics, as well as their work with mathematical spreadsheet modelling activities. There were clear differences between the two groups of students in their preference for external representations, in their understanding of the kind of answers they were expected to produce and in the way they conceived the role of mathematics in the practice of science. Although students' preferences for a particular representation were not significantly modified by the use of a spreadsheet as a modelling resource, at the end of the study the students recognised the value of using a more diverse set of representations. The results obtained suggest the possibility of enhancing students' capability to shift between a wider range of representations, including graphical, algebraic and numeric ones, using a modelling approach embedded in a computer environment such as a spreadsheet.

1. INTRODUCTION AND BACKGROUND

Mathematics provides one of the most powerful means of modelling and solving problems across a range of subjects in science and technology. Biology, chemistry and physics are all examples of subjects which make use of mathematical techniques to model situations and solve problems. Many of the mathematical techniques involve the use of methods which are considered valuable because of their generalisability. But it is this potential generalisability which is being questioned by the culture and cognition research community (for example Lave, 1988; Rogoff and Lave, 1984). Whereas there have been a number of studies which have investigated the ways in which mathematics is used in everyday practices (for example, Nunes et al., 1993) there has been very little work taking a similar perspective with respect to school practices such as science. Whatever the reasons, many students who study science cannot readily use mathematical methods within science subject areas.

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Investigating the mediating role of spreadsheets for representing and solving problems in science was one aim of a collaborative Mexican/British project (Sutherland et al., 1995). One of the main ideas in this project was the use of mathematics as a modelling tool. We worked with two groups of 16–18 year old science students and before introducing the spreadsheet as a medium for expressing and exploring mathematical models in science, we carried out extensive classroom-based observations of the ways in which mathematics was used in the practice of science. We always had the intention of investigating the cultural influences on these practices but when we started the project we did not anticipate the extent to which the pedagogic approaches in Mexico and the UK would impinge on actual mathematics-in-science practices.

The idea of structuring resource was central to the study. This drew on the work of Lave (1988) and allowed us to ask which resources students chose to draw upon when solving a particular problem and how these resources structured a problem-solving situation. Resources could include social resources such as the teacher and other students; semiotic resources such as language, graphs, algebra and tables; and physical resources such as calculators.

Within the project we have taken a sociocultural approach to mind, that is an account of human mental processes which recognises the essential relationship between these processes and their cultural, historical and institutional settings (Vygotsky, 1978). From this perspective the person-acting-with-mediational-means becomes the focus of analysis (Wertsch, 1991).

2. Research Method

Within the project we worked with Mexican and English students who were studying a two year course at school, prior to University entrance. In Mexico, students study a broad range of subjects for the whole two years, including aesthetic activities e.g. theatre studies and languages. All the Mexican students taking part in our project also studied physics, chemistry, biology and mathematics. The tradition in English schools is for students to choose a limited number of subjects to study after their national examinations at age 16 years (although this is currently under review). Students normally choose two to four ‘A’ level subjects which they study for two years, culminating in a further national examination. The English students in the project were studying three A-levels, predominantly chosen from biology, chemistry, physics and mathematics although some students were studying other subjects, for example, history. Both schools were innercity schools with an ethos of encouraging their students to be independent thinking and active people.

The project was designed to allow a diversity of research data to be collected and was structured into two main phases. The first phase involved a study of mathematical practices in school science, the second phase continued this study but involved the introduction of spreadsheet modelling sessions into the science classrooms. Over both phases, a series of individual student interviews were conducted to support observational and paper-based data collection and analysis. A summary of the data analysed is given below:

- *A study of mathematical practices in school science.* This included analysis of Mexican and English text books, curricula and syllabi; classroom-based observations carried out by the research team; analysis of students' written class and examination work; analysis of students responses to a written test of science problems which required mathematical solutions (a pre-evaluation).
- *A study of spreadsheet modelling sessions.* The Mexican and British studies were run in parallel and both the English and Mexican students worked on 6 common modelling problems within biology, chemistry and physics, together with a number of others, which were developed to fit in with the ongoing science curricula. Data collected included: video data of students; students' paper-based and computer-based work; researchers' notes.
- *Individual interviews with case study students.* Whereas the classroom observations and modelling activities were carried out with all the students in each class (9 students in Mexico and 26 students in England), we developed detailed case studies for 9 Mexican students and 12 English students. Only 7 of these English students also studied mathematics. The interviews were carried out at the beginning, middle and at the end of the study. The interviews centred on semi-structured questioning of the students on their responses to a range of paper and spreadsheet-based tasks.

The Mexican and UK research teams worked in a collaborative way to build up the theoretical framework, to design the modelling activities and to carry out the data analysis. The data collected were fully analysed for the two groups of students with members of each research team collecting data in both countries and co-analysing parts of the data to ensure common understandings were developed.

In this paper, we focus on the question, 'what resources do students draw on when working on mathematical problems within science, both on paper and working with spreadsheets ?' One way to address this question is to analyse their ways of working by focusing on their use of external representations, such as formulae, graphs and tables. However, this analysis must be considered within the context of the school mathematics and science cultures of the two countries. Thus in Section 3, we describe classroom cultures of science and mathematics in both countries. In Section 4, we present an analysis of students' mathematical problem-solving activity within science and report on results from the written pre-evaluation and initial interviews. In Section 5, we use the example of the *Diffusion* model and results from the 2nd interviews to illustrate the ways in which students used the spreadsheet to construct a model and draw on the representations as resources to make sense of the physical phenomena. We conclude (Section 6) by reflecting on how school mathematics culture influences students' use of external representations in solving science problems, and how working with spreadsheet modelling activities enables students to use and value a more diverse set of representations.

3. CLASSROOM CULTURES

In this section we describe the 'classroom cultures' in which the English and Mexican students worked. We conducted extensive classroom-based observations in both countries, including observations of classes in each others' countries to ensure consistency in data collection procedures[1]. The main aim of the classroom observation study was to uncover the extent to which mathematical concepts and methods were present in school science and to understand how mathematics was practised by students within school science situations. It quickly became apparent that the classroom cultures in the Mexican and the English schools studied exhibited significant differences, for example in terms of contrasting teaching approaches, and that these cultures influenced the ongoing mathematical practices.

In the Mexican school participating in the study, science was presented using what we have characterised as a *top-down* approach, in which general rules are emphasised and these are then used as a basis for approaching particular cases. The teachers worked from the front of the class making extensive use of the chalkboard for presenting and discussing scientific ideas. Dialogue between students and teacher usually focused on discussion of this written text and possibly more writing to elaborate a point. Text books were not usually used within the classroom situation, although students were expected to make use of the library as a resource.

In the English school, science was presented using a *bottom-up* approach in which students worked first on particular examples and were expected to use these to induce general theories. In the case study school, text books were extensively used as resources in physics and chemistry with students working through the text book in order to learn science (for example, the *Chemistry Students Book*, Nuffield, 1994). These text books also tend to use a bottom-up approach to presenting information and often introduce science within an ‘everyday’ situation using examples drawn from contexts outside of school. For example, the chemistry textbook, uses the ‘making of ginger beer’ to approach the chemistry of alcohols; the question ‘how does a pressure cooker reduce cooking time?’ is used to introduce the topic of rates of reaction. In general, teachers begin topics with reference to ‘everyday’ examples, rather than starting with the scientific concepts or more abstract ideas. The chalkboard is not used for the same presentational purposes as in Mexico and on some occasions was not used at all during the course of a science lesson.

Practical science activity in the Mexican school is separated from the usual classroom work, which is reserved for theoretical treatment of topics, and involves a different set of teachers. This separation of theory and experiment did not occur in the English school, where a science lesson normally contained a mixture of work. Evidence for differences in approach to practical work was first gained from analysis of the rhetoric of the science curricula. Where experimental work is mentioned in the Mexican curriculum, it is specified within the syllabus content, under instructions to the teachers on what to do and when. The syllabus language relates to the verification of already-learnt items, for example from the physics syllabus, ‘verifique los movimientos uniforme en trayectoria curvilinea y uniforme en trayectoria rectilinea’ (verify uniform curvilinear and rectilinear motion). In contrast, experimental work is viewed as an integral part of science in the English curriculum, with the expectation that students will engage in investigative work and develop techniques of hypothesis testing.

Our study of classroom cultures in science can be usefully combined with other work which has considered differences between approaches to mathematics in Mexican and English mathematics classes (Rojano et al., 1996). It seems that the mathematics courses experienced by the Mexican students also use a top-down approach in which general rules and algorithms (for example, the ‘*rule of three*’) are emphasised. Mathematics is predominantly taught without reference to non-mathematical contexts and thus there is very little emphasis on solving problems of a ‘scientific’ or ‘everyday’ nature. The mathematics course taken by the English students tends to be bottom-up with students working on specific examples and often being expected themselves to work out the general rules. Investigational work is also an integral aspect of students’ experience. There is considerable emphasis on work with graphs, often using graphic calculators, which is currently not the case in Mexico. In England there is relatively little emphasis on manipulative algebra (Sutherland, 1998) which is again in opposition to the Mexican case. However notions related to estimation are introduced in the early primary school in England which contrasts with many other countries.

We do not want to suggest that all Mexican school science and mathematics is approached in a top-down way and that all English school science and mathematics is approached in a bottom-up way. But we will argue in the next section that the characteristically different approaches observed in this study did impinge on the type of mathematics which students were able to draw on and the ways in which they used mathematics within their science practices.

4. Mathematical practices in school science: the pre-evaluation

Mathematics in science is predominantly used for expressing relationships between physical objects. How this is done varies between the sciences with physics making extensive use of algebraic representations and biology tending to make more extensive use of graphical representations and tables. These external representations become new tools to think with and manipulate. What is often implicit in the use of mathematics in school science is the link between a mathematical representation and the physical situation being represented. That students develop a facility to make links between the physical and the mathematical is essential for their success in science.

In addition to the observational study of classroom practices, we presented all the students with a written pre-evaluation which was designed to probe their approaches to solving problems involving the use of mathematics. The evaluation items included problems students were likely to encounter in their science studies and a selection of students were also asked to carry out some of the questions again in an interview situation. Here, we will consider a particular question which made use of three forms of representation to describe one kind of physical phenomenon, that of car motion. The presence of three mathematical representations within this question makes for a valuable research probe.

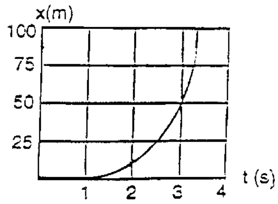
We found a strong set of related differences in students' approaches to this question, for example, a proclivity for either precise or estimated answers. Consider students' answers to question 1, Car B (What is the position of each car after 2 seconds?) and question 2, Car B (What are the individual positions of the cars at 2.5 seconds?) which both require stu-

Three cars A, B and C are travelling along:-

Car A

Time (seconds)	Position (metres)
0	60
1	70
2	75
3	70
4	60

Car B



Car C $x = 10 + 20t$

1. *What is the position of each car after 2 seconds?*
   Car A ....
   Car B ....
   Car C ....

2. *What are the individual positions of the cars at 2.5 seconds?*
   Car A ....
   Car B ....
   Car C ....

3. *At what time will each car reach position $x = 80$ metres?*
   Car A ....
   Car B ....
   Car C ....

4. *Describe what happens to Car A between t=2 and t=4 seconds.*

Figure 1. Question used in Pre and Post evaluation.

dents to read from the graph. The majority of Mexican students answered question 1, Car B, by writing 12.5 m and the majority of English students answered either 12 m or 12.5 m, however their ways of working and verbal explanations differed as we shall subsequently explain. The vast majority of English students answered question 2, Car B as 25 m, which was a sensible answer whereas half the Mexican students gave an inappropriate answer of 31 m or 31.25 m, which related to their application of inappropriate techniques as discussed below.

When interviewed, several of the Mexican students explicitly pointed out that their answers were not precise when using the graph, (indicated through the use of symbols which signify approximation or by writing 'approx.' and verbally stating their difficulty in obtaining a precise answer) and they were not comfortable with having to write down approximate answers. One student who answered the second question by writing 31.25, explained when interviewed how she worked out the number 'I considered the 2.5 and I went to the y-axis in order to see where it was . . . it is very difficult to say . . . I tried to divide this into several equal parts'. This explanation suggests a concern with producing an answer which is exact. She explicated the difficulty of identifying a rough value and decided to use a finer partition of the interval ('I tried to divide this in several equal parts') in order to give an answer which was as precise as possible. Some of the Mexican students drew vertical and/or horizontal lines corresponding to the abscissa/ordinate of the point on the graph in an attempt to gain accuracy.

In contrast to the Mexican students, the English students were comfortable with providing estimated answers with most of them simply 'eyeballing' the graph to obtain an answer. Interview responses illustrate this approach, for example, 'you look at B and that has got a graph and then you find 2 seconds and then you try and estimate what it is by going across so you know half the square is about 12.5. I normally see where it is, if it is above, under or over the half way mark so it is about here or there'. Similarly, 'I looked at car B and after 2 seconds it was about . . . like I had to work out half of the 25 and then just estimate where the line was'. The language of estimation is clearly evident and contrasts with the Mexican students search for strategies to allow increased accuracy.

Mexican students tended to draw on algorithmic methods to solve science problems, for example 'the rule of three', and used these rules as resources even when interpreting tables and graphs. This is typified by one student who stated in interview a preference for symbolic representations rather than tables or graphs and frequently used the 'rule of three', even in cases when it was not appropriate. During one interview she explained the process she followed for answering question 2, Car A: ‘I used the rule of three . . . as 2 is to 75 as 2.5 is to x, and I got the quantity’ (‘. . . lo que hice fue una regla de tres, tan sencillo como 2 es a 75 como 2.5 es a x, y me salió esa cantidad’). The use of such a taught algorithm seems also to be related to a need to provide a precise answer, as discussed above.

As well as a differing emphasis on precise or estimated answers, students exhibited different approaches to the algebraic representation in terms of ‘one’ or ‘many’ answers being possible. For example, in response to question 1, Car C (see Figure 1) Mexican students viewed the equation $x = 10 + 20t$ as being an equation with ‘one answer’ whereas the English students tended to think in terms of many answers ‘You can put any number into it and get an answer’, perhaps suggesting a more dynamic, as opposed to static, view of equations. The English students tended to make use of more informal methods as resources such as ‘trial and refinement’ when solving equations. For example, to answer question 3, Car C, they would insert an estimated value for ‘t’ into the equation and use a calculator to work out what value of distance ‘x’ this produced. They would then revise their value of ‘t’ and evaluate this answer. This process would be repeated until a value for ‘t’ which had the outcome of $x = 80$ was obtained. This contrasts with the Mexican students’ approach of using a formal algebraic manipulation strategy.

We believe that all the differences in approach exhibited by the two groups of students can be explained in terms of the cultures described in the previous section. For example, we suggest that the more dynamic view of an equation described above relates to the emphasis on functions and graphs in the English mathematics curriculum, whereas the Mexican approach relates to the emphasis on algebra as a means of solving equations. Students’ choice of resources used for answering questions is influenced by their desire for precision, or contentment with estimates, arising from what is emphasised, and the teaching approach, in the different classroom cultures.

To summarise, analysis of the pre-evaluation, supported by evidence from the observation of classroom practices and initial interviews, showed that there were clear differences between the two groups of students in their responses to paper-based problems. There were differences in the preference for the resources they chose to use and in the understanding of the kind of answers they were expected to produce. Alongside this, there were strong similarities between students within each group. For example, all Mexican students were concerned with producing exact answers, they made an extensive use of taught algorithmic methods and their interpretation of formulae tended to be disconnected from the scientific situation represented. In contrast, English students were comfortable with approximate answers, tended to use informal methods, for example 'visual estimation', and their interpretation of formulae was dominated by the scientific situation being represented. Our view is that these differences and similarities find origin in the different school cultures which the two groups of students experienced.

5. MATHEMATICAL MODELLING WITH SPREADSHEETS

One of the motivations to carry out a project on mathematical modelling with spreadsheets in science was to try to overtly bridge the gap between mathematics and science in the classroom. Our starting point centred around the idea that the different representation systems in mathematics (such as graphs, numerical tables and formulae) allow an analysis of different aspects of phenomena and that a spreadsheet (which involves all of these representations) could thus facilitate the analysis of science problems. An 'algebraic' approach to modelling science situations separates out the physical context and places emphasis on the mathematical manipulations needed to solve a problem. However the format of the spreadsheet retains aspects of the scientific problem, for example, with the labelling of columns. The transformation of formulae into lists of numbers and graphs would confront students with a need to work with a variety of representations and with the need to make links *between* representations.

The term mathematical modelling has been used with many different meanings but in essence involves movement between a physical situation being modelled and the mathematical representations of that model (Mason and Davis, 1991). Ogborn considers modelling as 'thinking about one thing in terms of simpler artificial things' (Ogborn, 1994). Based on this idea, he defines a model as an artificial world with the characteristic that all its components are known, since when building it up it was decided what these components were meant to be. We consider that this is one of the strengths of models: it is already known which elements or variables of the phenomenon are taken into account and which are being left aside. In this 'artificial world', the user can express her (his) own ideas about a situation (expressive modelling) or can explore others' ideas (exploratory modelling). In this project, the majority of the modelling activities were exploratory, which means that some or all of the components of a model were presented to students, the student built up a spreadsheet version and used this spreadsheet model to explore different aspects of the physical phenomena or situation being modelled. For example, a spreadsheet allows students to change the values of the parameters involved and to see immediately the effect on the tables and graphs, giving them the power to analyse many cases with a simple change of a number in a cell.

Students in both countries worked on a range of modelling problems, for example diffusion in biology, chemical equilibrium in chemistry and collisions in physics. The Mexican students had approximately 21 hours of introductory spreadsheet work situated within computer lessons and 12 hours of spreadsheet modelling within science lessons. The UK students had approximately 5 hours of introductory spreadsheet activities within the science lessons, and 17 hours of spreadsheet modelling within science lessons. Students usually worked in pairs, using portable computers which they were able to borrow in order to complete their class work. After constructing a spreadsheet model, the students were asked to construct graphs relating the variables of the model and to explore the situation by changing various parameters corresponding to different physical situations (Sutherland et al., 1996; Rojano et al., 1996).

Neither of the groups had experienced mathematical modelling before the study, although a sub-set of the English students (who were also studying post-16 mathematics) were being introduced to modelling in mathematics. In both Mexico and England, before the spreadsheet activities, most students had difficulty articulating their understanding of a mathematical model and several professed to have never heard about such an idea (evidence from the initial interviews). An example of the modelling situations we designed is given in Figure 2 which shows the front page of the *Diffusion Model* activity, used with both groups of students.

The physical situation was presented first, using text and diagrams, with students then being asked to conduct some numerical calculations to complete a table. The idea was to engage the students in the physical situation, to support them to work out the mathematical model being proposed and to generate the underlying general formula. The intention was that students would use this initial activity to support development of their spreadsheet model and subsequently use the different systems of representation in the spreadsheet to develop their understanding of the phenomenon.

Figure 3 shows the table filled in by one of the Mexican students and illustrates the construction towards a general model through a series of specific calculations.

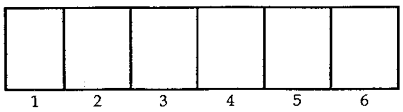
Passing from the data table produced with a calculator (Figure 3) to one produced using general formulae on a spreadsheet enabled students to move from the particular to the general and from the physical situation to a mathematical model. The formula which produced the data could be immediately accessed by students (by clicking onto a spreadsheet cell) whereas the mechanism used to produce a number on a calculator is usu-

A MODEL FOR DIFFUSION IN A CELL

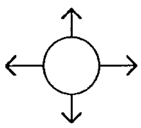
In this activity you will use a spreadsheet to develop a model of the diffusion process.

Background to the model

Below is a cell, split into 6 imaginary compartments and numbered 1 to 6 for convenience. The cell could be in any living organism.



Cell wall



Molecule and movement directions.

In the left hand side compartment, there are 1200 molecules. The other compartments are empty to begin with.
Nothing can move in or out of the cell.
Each of the 1200 molecules can move up, down, left or right and they are equally likely to move in any of these 4 directions.
As the cell is small, in one movement a molecule can reach the top or bottom of the cell, or another compartment on either side (note that the end compartments will be different). Assume that each molecule makes one move in one time period, say one second.

In order to help you to start understanding this model, work out how many molecules will be in each compartment initially (t = 0), after 1 sec. and after 2 sec. and fill in the table.

time/ sec	*comp.* 1	*comp.* 2	*comp.* 3	*comp.* 4	*comp.* 5	*comp.* 6
0						
1						
2						

You can use a spreadsheet to set up the model and follow the movement of the molecules with time. This will give you a picture of what happens to molecules as they diffuse inside a cell.

Figure 2. Front page of the Diffusion Model activity.

In order to help you to start understanding this model, work out how many molecules will be in each compartment initially (t = 0), after 1 second and after 2 seconds, and fill in the table.

43.75

112.5

356.25
- 18.75

time /	comp. 1	comp. 2	comp. 3	comp. 4	comp. 5	comp. 6
0	1200	0	0	0	0	0
1	900	300	0	0	0	0
2	750	375	75	0	0	0
3				11.75		
4						

Figure 3. Data produced by a Mexican student using a calculator.

ally not visible. Having created a data table from inputting spreadsheet formulae, students went on to produce graphs of the number of particles in compartments, over time. Thus students themselves constructed the links between numeric, symbolic and graphic representational forms.

All of the students had the opportunity to work with all the representations in the spreadsheet, using formulae to express relationships between variables; tables to display variation numerically and graphs to verify intuitive predictions and to get a ‘global’ view of the phenomenon. Although the models had different characteristics, all the activities made use of tables, graphs and formulae, with associated questions to provoke students into more spontaneous use of the representations. For the students the spreadsheet became an important tool not only to perform calculations, but also to give structure to the modelling process and to provide feedback on the model $<->$ phenomenon links. For example, analysis of video data indicates that the spreadsheet acts as a structuring resource when identifying the variables within a model, ‘This column is going to be (pointing to the column) the amount of substance ... at a time ... so N is the amount we came out at the end ... so it’s that which is N_o, which is 100!’.

In the second interviews, after all students had engaged in four modelling activities, we probed the role of spreadsheet activities in their understanding and conceptualisation of scientific phenomena. Students’ previous work with a specific model was used to frame the interview. In Mexico a model on chemical equilibrium was used, with a model on collisions and momentum used in England[2]. We analysed students’ use of different resources as they answered questions on specific aspects of the scientific phenomenon and as they used representations in the spreadsheet model they had produced. Some of the students gave reasons for their preferences for a particular representation, for instance, in the Mexican group, Juan stated ‘graphs are not as logical as numbers’, he understood the graph only if he had made sense of the table first. Eduardo justified his preference for formulae with, ‘formulas make it easier to work with so much data’, and they helped him to ‘determine all the values representing the problem’. Marina, who claimed to prefer formulae in the first interview, showed in the second interview how she valued the graphical representation in the chemical equilibrium model, ‘they (the graphs) helped me to know the proportions of the chemical reaction’.

When Mexican students were asked what they understood by chemical equilibrium and why it is called a ‘dynamic equilibrium’, all used spreadsheet representations to give their explanations. For example Litzli, spontaneously used her graph to answer questions about the behaviour of the reaction over time:

I: So, what will happen finally?

L: We see the graphs . . . what they are telling us is that with time, there will be a moment at which it is going to be constant . . . the number of balls in one receptacle will be the same as in the second one. . . . Although they are exchanging (balls) they won't vary . . .

I: Where can I see this?

L: Well, in the graph, that is . . . each line represents one of the balls from the right and one from the left, and then . . . we see that the lines are parallel . . .

Juan, another Mexican student, chose to use the spreadsheet data table to locate the time at which chemical equilibrium was reached. He said, 'it is reached when the amount going into a system and the amount coming out, move at the same rate'. Marina also used the data table and the graph to explain her notion of a dynamic reaction, 'even if it is in equilibrium, it keeps moving'.

In the English group, students also used a mixture of representations to interpret questions on the collisions model. Adam, for example, used a graph to link the physical situation with the mathematical model and to draw inferences about change in parameters. He described various situations concerning two balls colliding – such as when the balls are of differing mass – with reference to the spreadsheet graph of his collision model,

A: Its got bigger values, it's going faster. . . . When that ball has a mass of less than the first ball, that means it's not going to move, it's going to bounce off it. And when it has a bigger mass, it's going to push the other ball as well.

However, he appreciated that the data table was a more appropriate resource in other ways, for example, when needing to find the post-collision velocity when ball masses are equal,

A: [a graph is] easier than having a load of numbers in a table[but] it's probably easier to use the table . . . because you just go down to where the mass is the same, then across . . .

As the project progressed, the majority of Mexican students began to use graphical representations in the context of spreadsheet modelling despite their expressed preference for algebraic representations (1st interview) and tables or formulae (2nd interview). In a similar way, the majority of English students began to use algebraic representations (spreadsheet formulae) despite their preference for graphical representations. We believe that this move towards a more diversified use of external representations was provoked by the role that each of these representations played in the modelling activities. The representations acted as structuring resources in the mathematical thinking about a physical phenomenon.

We found that even though the modelling activities were planned in a similar way in both countries, and in some cases were identical (although in English and Spanish), the school mathematics culture influenced the way students worked with a cognitive tool in which a diverse range of representational resources were available to represent and interpret physical situations.

6. CONCLUSIONS

We have found many examples of school mathematics structuring the mathematical practices of students in science, and differences in approaches between the Mexican and English students can often be traced to differences between school science and mathematics cultures. It also appears that the representations prioritised by a teacher are influenced by the teaching approach. A more presentational style of teaching seems to be linked to an emphasis on precise answers. The emphasis on precision seems to be supported by algebraic representations. A more exploratory style seems to promote approximation and to be linked to an emphasis on graphical representations.

The results obtained in this study serve to illustrate the advantages and disadvantages of different types of representation as resources for solving scientific problems and indicates that no one approach is sufficiently rich to ensure a complete analysis of the different aspects of a phenomenon. The facility to shift between a range of representations including, graphical, algebraic and numerical ones, is a powerful skill for students.

Our work suggests that mathematical practices within science subjects may be influenced through the use of a modelling approach embedded in a computational environment such as a spreadsheet. The spreadsheet approach supported Mexican students to appreciate and use graphical and numerical representations and at the same time supported English students to make sense of algebraic representations. It is possible that such spreadsheet modelling approaches will thus afford a route to accessing the facility of widened representational use.

What we have focused on in this paper is the interrelationship between cognition and culture as emphasised by Bruner (1996) and Wertsch (1991). This perspective emphasises the fact that mind cannot exist without culture, with culture being mediated by language and other semiotic systems. We have highlighted the differences between school mathematics cultures in Mexico and England and shown how differences in students' approaches to solving problems reflect differences in school culture. Ours was not a comparative study in the traditional sense, but rather two parallel case studies of two educational systems for pre-university students. As Bruner has pointed out 'to take a cultural view of education does not really require constant cultural comparison. Rather, it requires that one consider education and learning in their situated, cultural context.' (Bruner, 1996)

NOTES

[1] Both Mexican and British teams visited the other team twice during the study. This was a crucial part of the collaborative process.

[2] In the chemical equilibrium model, the number of balls in two receptacles simulated the concentration of molecules in initial (reactant) and final (product) states and students were asked, for example, to study the effects of reactant concentration on the system. In the collisions model, people on an ice rink and billiard balls provided the contexts to study inelastic and elastic collisions. Students were asked to investigate, for example, the effect on collisions of varying the mass of one moving ball.

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