COGNITIVE TENDENCIES AND THE INTERACTION BETWEEN SEMANTICS AND ALGEBRAIC SYNTAX IN THE PRODUCTION OF SYNTACTIC ERRORS

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In Filloy and Rojano (1989) we introduced the use of concrete models for the teaching of solving linear equations and studied the abstraction processes that take place when such models are put in work by 12-13 year olds. In this paper we discuss M and V cases where, on the face of elements provided by the same "concrete" model for the operation of the unknown, the evolution paths of their use for the resolution of more and more complex equation modes are dissimilar (in fact, antagonic). However, in spite of this antagonism there is a common tendency to abbreviate the processes involved and this generates (in both cases) a number of well known algebra learning obstacles and syntactic errors.

The literature about algebraic errors in the learning of algebra is mainly focused in its syntactic component: Matz (1982), Kirshner (1987), Drohuard (1992). Few works like Booth (1984) and Bell (1996) situate this problematic component in a more general context, for instance, problem solving. In this paper we analyse the interaction between semantics and algebraic syntax as a source of syntactic errors, when this ineraction takes place in teaching processes that involve concrete modeling. We argue that such analysis provides a perspective that allows us to give different explanations of the presence of some typical algebraic syntax errors.

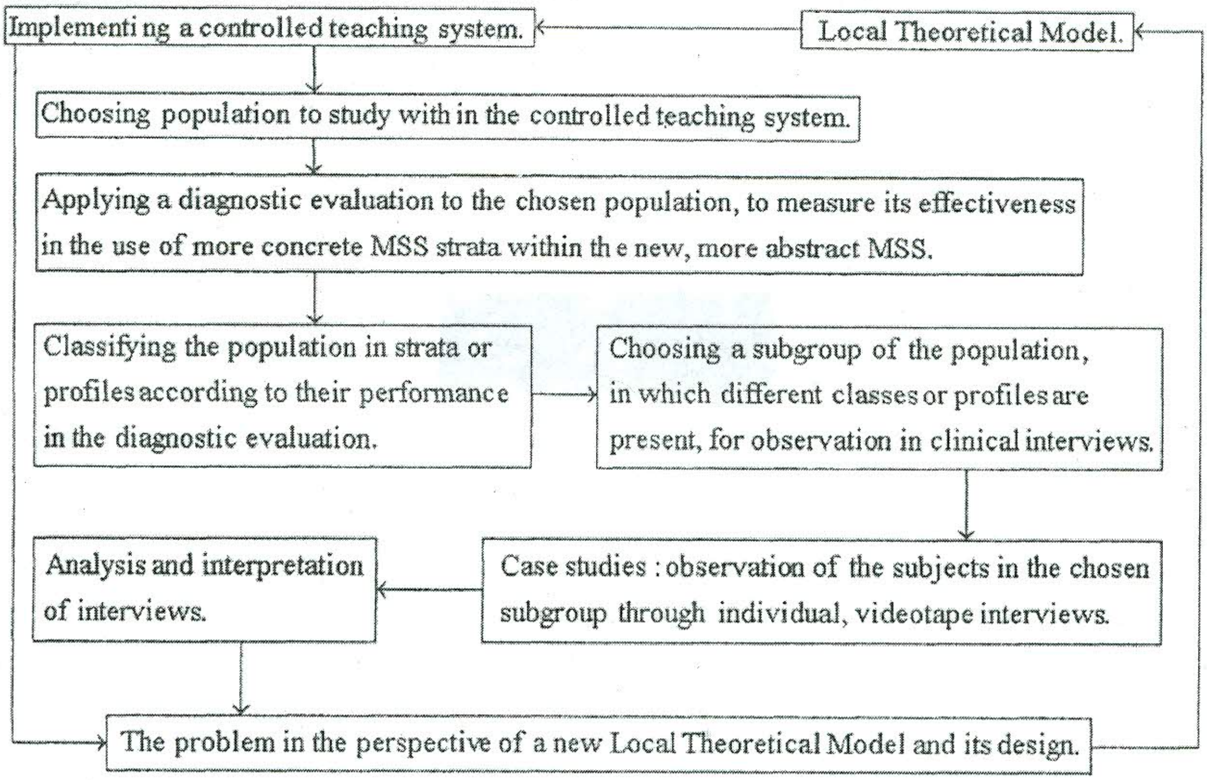
Undertaking a semantic introduction to new algebraic concepts, objects, and operations implies selecting a concrete situation (i.e., a situation which in some context is familiar to the learner) in which such objects and operations can be modeled. With this approach it is possible to resort to previous knowledge, in order to accomplish the attainment of new knowledge. This is one of the driving principles of modeling, the strengths and weaknesses of which become manifest at the time a specific model is put into operation (see Filloy/Rojano, 2001, for a more detailed description). In the cases we report here (V and M cases) the concrete situation in question is a geometric model that was used as a semantic introduction to the operation of the unknown for the resolution of the first non-arithmetical equations. In this model, the translation of the proposed equation into equalities between quantities or magnitudes in a more 'concrete' situation permits to find out the numnerical value of the unknown in the context of area comparison. The use of such a geometric model, then, presupposes a good handling of operations with areas. This handling, as can be verified in V and M's interviews, is a requirement that was covered in both cases. When this study was carried out, M and V showed to be highly proficient at school maths. They found no difficulties in handling the model during the instruction phase aimed at modeling the first non-arithmetic equations (equations of the form $Ax+B=Cx$, where A, B, and C are given positive integers, and $C>A$). It is in the transition to more complex modes of equations that modeling and actions in the model, in turn became more and more complex. In contrast to previous explanatios given with regards pupils' syntax errors, within the modeling realm it is possible to formulate explanations grounded on the nature of the model and the sort of cognitive tendencies displayed by the subjects.

THEORETICAL AND METHODOLOGICAL FRAMEWORK

In Filloy (1990) we introduced the methodological framework of **local theoretical models** in which the object of study is brought into focus through four inter-related components:

(1) **Teaching models** together with (2) **models of cognitive processes**, both related to (3) **models of formal competence** that simulate competent performance of the ideal user of a Mathematical Sign System (MSS) and (4) **communication models** to describe rules of communicative competence, production of texts, texts decoding, and contextual clarification.

The following scheme describes the rationale of the case study:



Results from the diagnostic test located V and M in the category of students with high proficiency in a) solving arithmetic linear equations; b) solving arithmetic word problems; and c) numeric skills. Before the interview, V and M had not been introduced to the learning of algebra. Once V and M got to solve the first non-arithmetic equations by means of the geometric model, they were faced with more