Arithmetic World - Algebra World

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The power of the Cartesian method is that expressing as an equation the elements of a word problem permits to reach the solution by solving such equation at a mere syntactic level, without making reference to the problem context. This is an idea that the majority of secondary school students find extremely difficult. Results from the Anglo/Mexican project *Modelling algebra problems within a spreadsheet environment* (1) suggest that using a spreadsheet method, pupils are able to express the relationships of the problem and to search the solution through numerical means. Carmen's case illustrates these results and shows the feasibility of connecting a spreadsheet approach with the Cartesian method.

Introduction

In spite of the changes produced in school algebra by mathematics curriculum reforms in many countries, students' use of the Cartesian method still remains being an important goal at this school level. Nevertheless, a number of studies indicate that junior secondary school pupils are more likely to use non-algebraic methods when solving word problems (Bednarz *et al.*, 1992; Bednarz et al, 1996 Lins, 1992). This has led research to focus on the analysis of pupils' strategies with the aim of probing the nature of their informal non-algebraic approaches. The Anglo/Mexican Project deals with other aspects of solving problems in algebra. This project was developed to help students bridge the gap between arithmetic and algebraic thinking alongside two evolution lines: 1) basic algebraic concepts and 2) problem solving methods. The main aims of this project were to:

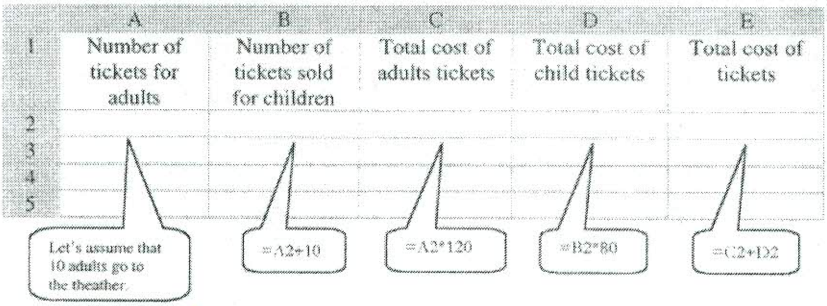
- Investigate the way in which pupils use a spreadsheet environment to represent and solve algebra problems relating this to their previous arithmetical experiences and their evolving use of a symbolic language.
- Characterize pupils' problem-solving processes along the dimension arithmetic/algebraic as they evolve through working in a spreadsheet environment.

We worked with two groups of eight 14-15 year olds (one in Mexico and one in Britain) who had had a history of being unsuccessful with school mathematics. In a pre-questionnaire, these pupils showed to be reluctant to use school methods. In particular, some of them showed to be reluctant to use algebra methods to solve word problems. Pupils of the study were engaged in spreadsheet activities, which focused on the notion of function and inverse function, equivalent algebraic expressions and the solution of algebra word problems. They used a spreadsheet cell to represent the unknown (entering a numeric value in the cell) and then with the spreadsheet code, produced a formula to express algebraic relationships in terms of this cell (see, for example, Fig 1 *The theatre problem*). This way of dealing with unknowns, both in a symbolic and in a numeric way, allowed these pupils to make a step in accepting the idea of operating with an unknown quantity. According to Filloy & Rojano (1989), this idea represents an obstacle that secondary students have to overcome, in order to have access to the realm of algebra. Operating with unknown quantities, in turn, constitutes the core of algebraic methods for solving word problems.

The theatre problem.

Tickets for a theatre performance cost $120 for adults and $ 80 for children. A hundred tickets more for children than for adults were sold. How many tickets for adults and for children were sold if the total collected amount was $30 000 ?

Use the spreadsheet to solve this problem.



How much money will be collected if ten adults go to the theater? $ ______________

Change the number (tickets for adults) in cell A2.

How many tickets for adults were sold? ______________

How many tickets for children were sold? ______________

Figure 1. *Spreadsheet Method to solve word problems (Worksheet used in the Anglo/Mexican Project, here we write down the formulas that pupils are expected to enter in the cells).*

Results from the Anglo/Mexican project, which focus on pupils' conceptual development in algebra have been synthesized in previous papers (Sutherland & Rojano, 1993; Rojano & Sutherland, 1993 and 1994; and Rojano, T., 1996). Results concerning methodological aspects of the transition towards the algebraic realm suggest that it is feasible to support pupil's to switch from informal strategies to algebraic methods of solving word problems by means of using a spreadsheet to express the relationships between the elements of the problem. An analytic tool derived from the *mathematical analysis and synthesis process* was used to interpret children's productions (Rojano & Sutherland, 1997; Rojano, to appear). In this paper, we discuss the case of a 15 years old Mexican girl from the group of the algebra-resistant pupils, who got to link the use of spreadsheets to express the state of affairs of a word problem with the algebraic code. This case exemplifies a possible way in which pupils can conciliate the domain of manipulative algebra with the process of analysis, which is implied in the method of solving word problems.

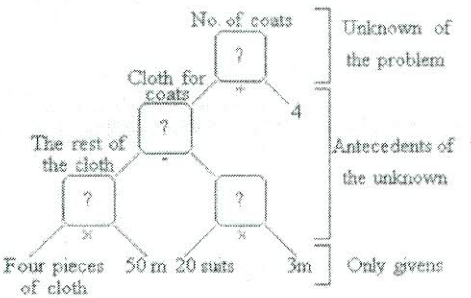
Arithmetic methods - Algebra methods

Puig and Cerdán (1990) develop an analysis of the **translation processes** of the problem statement into an arithmetic or algebraic expression. These authors use as tools of analysis two general methods: **the method of analysis and synthesis** and **the Cartesian method**. The first one (the method of analysis and synthesis) leads to a translation process of an arithmetical nature, which consists of transforming the initial text of the problem into a new text, in which the elements that intervene in more elementary translations are made explicit, in order to make explicit, as well, the way these elements are linked within the arithmetic expression that solves the problem (Puig and Cerdán, 1990, pp. 38-39). The intermediate texts produced in this process involve intermediate variables or unknowns, which are called *the antecedents of the unknown* (Lakatos, 1978) and the idea is to produce only *givens* in the final step of this sequence of transformations. This is the *analysis process*, and the inverse one (to perform the operations with the givens to find the unknown value) constitutes the *synthesis process*. Puig and Cerdán use an *ad hoc* diagram to represent the *analysis process* (Botsmanova, 1972) which is illustrated in Figure 1.

The Problem

Four pieces of cloth of 50 m each will be used to make 20 suits, which need 3 m of cloth each. The rest of the cloth will be used to make coats, which need 4 m each. How many coats can be made?

The analysis process :



The synthesis process:

$$[(4\times50)-(20\times3)]\div4$$

Arithmetic Expression

Figure 1. *The intermediate text*

Puig and Cerdán (1990, pp. 40-42) give examples of problems that can not be reduced to an arithmetic expression via the *method of analysis and synthesis* but which when applying *the method of analysis* leads to an equation in terms of unknowns instead of an expression involving only givens. So, in these cases, the *synthesis process* is impossible. According to these authors, *in the limit of the analysis-synthesis, the method becomes algebraic when the unknown of the problem is considered as a given, useful to determine the unknown itself, that is,* ***unknown and givens are treated in the same way.***

The former is a way of characterizing word problems and solving methods as either arithmetic or algebraic and it was used as a tool of analysis to interpret pupils' productions in the Anglo/Mexican Spreadsheets Algebra Project.

Arithmetic / Algebraic Approaches

Children's strategies to solve algebra word problems give account of a solving approach, which proceeds from the known to the unknown. This approach is in opposition to that of algebra in which working with unknown quantities is in the core of the method. *The Chocolates Problem* was presented to the students in a pre and in a post-interview. The relationships between the unknowns are explicitly given in this problem and it is considered of a high degree of difficulty because it involves three unknowns.

The Chocolates Problem

100 chocolates are distributed amongst three groups of children. The second group receives four times the number of chocolates as the first group. The third group receives ten chocolates more than the second group. How many chocolates does each group receive?

An algebraic solution to this problem leads to a set of equations such as the following:

$$y = 4x$$
$$x = y + 10$$