ALGEBRAIC REASONING WITH SPREADSHEETS

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1. Introduction

For a long time now, students have been introduced to algebraic reasoning during secondary school. However, results reported in recent studies suggest that it is possible to introduce students to basic notions of algebra at an early age (8-10 years old students) (Carraher, Schlieman & Brizuela, 2001). This may have repercussions on the curriculum, in which there will probably be a more and more explicit presence of algebra from elementary school onwards. However, from the point of view of knowledge development being based on ruptures, access to algebraic thinking may be conceived as a group of transitions during which individuals must confront and overcome a series of obstacles or difficulties, regardless of the age or school level at which they are first introduced to this type of thinking.

Among recognised transitions towards algebraic thinking, both due to analysis from a theoretical-epistemological perspective as well as to empirical studies carried out during the 80s and 90s, we may find: the transition from that which is numeric or verbal to that which is symbolic; from that which is specific to that which is general; from working with that which is known to that which is unknown; from the application of intuitive methods (or non-algebraic) to the application of school methods (or algebraic). Therefore, from this point of view, educational approaches, which promise access to the powerful ideas of school algebra, would have to clearly state how they will endeavour to help students carry out such transitions. Examples shall be given in this article with regards the way in which the availability of computer environments, which possess a large capacity for numerical calculation, an algebra-like interactive language and which offer the possibility of an on-screen multi-representational display, will permit the creation of learning environments where students will be able to overcome difficulties inherent in the development process towards an algebraic way of thinking. For this reason, we shall refer to a series of investigations carried out with children aged between 10 and 16, in which spreadsheets were used in the design of classroom activities. This design is undertaken according to the basic principle that, when students begin to study algebra, they already have prior knowledge relating to mathematical methods of

reasoning, which is based to a large extent on that which is numerical, specific, intuitive and on that which is known (that which is given).

2. Arithmetic problems – algebraic problems

The work of Puig and Cerdán (1990) is one of many attempts to unravel the arithmetic or algebraic nature of a problem. However, in operating terms, it is still very difficult to establish a strict differentiation between arithmetic and algebraic problems. In this article, we simply consider algebraic problems to be those word problems whose analysis leads to an algebraic representation of the relationship between data and unknown quantities. Furthermore, in accordance with Puig & Cerdán's ideas, we shall add the specification that, during the course of such an analysis, unknown quantities must be dealt with in the same way as data. This last point relates to the distinction between arithmetic and algebraic methods of word problem solving, which is the central point of this section.

2.1 Arithmetic methods – algebraic methods

The main aim of F. Viète when he invented symbolic algebra was to provide mathematics with a language and tool for the solving of all problems (Viète, 1983). Nowadays, when we refer to this instrumental algebraic character, we are referring to the so-called *Cartesian method*. This method can be applied to the solving of word problems and consists of: an analysis of the elements of the problem and their relationships, formulation of equations for the relationships between data and unknown quantities as well as between unknown quantities, solving of the equation (or system of equations) in order to finally provide the solution (or solutions) to the context of the problem. This series of steps, which lead to the algebraic solution of a problem, is the method that secondary school students are normally expected to learn and apply. However, as has been shown in many investigations, school systems have not managed to fulfil this goal. The reason for this is that the application of this method requires natural language to be translated into algebraic code, which, in turn, presupposes a change from the arithmetic method of problem-solving (in which an analysis is carried out of that which is known (the data) in order to find the value of that which is unknown) to an algebraic method in which we have to start from the beginning as if the problem were already solved (Charbonneau, 1996). In other words, unknown quantities must be involved in the analysis process. This change, which is epistemological and which corresponds to a transition towards working with that which is symbolic and with that which is unknown, requires intensive guidance and intervention by teachers in schools as well as acceptance, by the student, of the required break with arithmetic methods (Filloy & Rojano, 1989; Balacheff, 2001). We will show below how spreadsheets can be used in the task of helping students change from intuitive, arithmetic methods to algebraic methods for word problem solving.

2.2 The role of spreadsheets in the transition towards the algebraic method

The following is an example of how the use of a spreadsheet can assist in the analysis process of a problem's statement.